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Duality and hidden dimensions*

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Abstract

Using a global superalgebra with 32 fermionic and 528 bosonic charges, many features of p-brane dualities and hidden dimensions are discussed.

1 Introduction

It is a pleasure for me to participate in the celebration of Keiji Kikkawa's birthday. Considering that Kikkawa and Yamasaki [1] were the first to notice T-duality, and the first to discuss membranes in the context of unification [2], it is only appropriate that I concentrate my discussion on duality in strings, membranes and more generally p-branes.

The discovery of string dualities [3] [4] have led to the idea that there is a more fundamental theory than string theory, "M-theory" [5] [6] [7], that

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manifests itself in different forms in certain regimes of its moduli space. The several familiar string theories (type-I, type-II, heterotic) may be regarded as different starting points for perturbative expansions around some vacuua of the fundamental theory, in analogy to perturbative expansions around the different vacuua of spontaneously broken gauge theories. A lot of evidence has accumulated by now to convince oneself that the different versions of $D = 10$ superstrings and their compactifications are related to each other non-perturbatively by duality transformations. Furthermore, there is evidence that the non-perturbative theory is hiding higher dimensions and that it is related to various p-branes [8] and D-branes [9].

Although I refer to "M-theory" I will not discuss it directly. Instead, without going into the details of string theory or M-theory, I will connect the duality and 11D (or even 12D) properties to a superalgebra involving 32 supercharges and 528 bosonic generators [8] . Therefore, I will begin my discussion by outlining some of the properties of the superalgebra and the interpretation of its structure. I will then discuss examples of how U-duality and hidden spacetime dimensions become manifest in the non-perturbative spectrum of the theory. For more details see [10] for the first part and [11] [12] for the second part.

2 Dynamical superalgebra and p-branes

It is well known that the maximum number of supercharges in a physical theory is 32. This constraint is obtained in four dimensions by requiring that supermultiplets of massless particles should not contain spins that exceed 2. Assuming that the four dimensional theory is related to a higher dimensional one, then the higher theory can have at most 32 real supercharges. The supersymmetry associated with these supercharges is not necessarily exact; it may be broken by central extensions included in the superalgebra. Denote the 32 supercharges by Q^a_α, where $a = 1, 2, \cdots N$, and α is the spinor index in d-dimensions. For example, in $d = 11$ there is a single 32-component Majorana spinor (N=1), in D=10 there are two 16-component Majorana-Weyl spinors (N=2), etc. down to D=4 where there are eight 4-component Majorana spinors (N=8). It is important to note that 32 corresponds to counting *real* components of spinors.

In 12 dimensions the Weyl spinor also has 32 components since $\frac{1}{2}2^{12/2} =$

32, but when the signature is $(11,1)$ the spinor is complex and has 64 real components. Therefore, as long as we consider a single time coordinate, $d=11$ is the highest allowed dimension. However, if the signature is $(10,2)$, it is possible to impose a Majorana condition that permits a real 32-component spinor. Beyond 12 dimensions the spinor is too large, and therefore we cannot consider $d>12$.

The 32 spinors Q_α^a may be classified as the spinor for $SO(c+1,1)\otimes SO(d-1,1)$ with $d+c+2=12$. Here c is interpreted as the number of compactified dimensions from the point of view of 10D string theory, and the extra 2 dimensions are considered hidden. This spinor$\times$ spinor classification is given in Table I for each dimension. The index a corresponds to the spinor of $SO(c+1,1)$. This group is not necessarily a symmetry, but it helps to keep track of the compactified dimensions, including the hidden ones. Furthermore, the same index a will be reclassified later under the maximal compact subgroup K of U-duality, thus providing a bridge between duality and higher hidden dimensions.

Consider the maximally extended algebra of the 32 supercharges in various dimensions in the form

$$\left\{Q_\alpha^a, Q_\beta^b\right\} = \delta^{ab}\gamma_{\alpha\beta}^\mu \ P_\mu + \sum_{p=0,1,\cdots} \gamma_{\alpha\beta}^{\mu_1\cdots\mu_p} \ Z_{\mu_1\cdots\mu_p}^{ab}. \tag{1}$$

Since the left side is the symmetric product of 32 supercharges, the right side can have at most $\frac{1}{2}32\times 33=528$ independent generators. The indices ab on $Z_{\mu_1\cdots\mu_p}^{ab}$ are either symmetrized or antisymmetrized and have the same permutation symmetry as $\alpha\beta$ in $\gamma_{\alpha\beta}^{\mu_1\cdots\mu_p}$. The central extensions $Z_{\mu_1\cdots\mu_p}^{ab}$ are assumed to commute with Q_α^a, P_μ, but they are tensors of the Lorentz group and hence do not commute with it.

In (10,2) dimensions we will use $M=0',0,1,2,\cdots,10$ for the space index instead of μ. In the 32$\times$32 representation (equivalent to chirally projected 64$\times$64) only the 2- and 6- index gamma matrices $\gamma_{\alpha\beta}^{M_1M_2}$ and $\gamma_{\alpha\beta}^{M_1\cdots M_6}$ are symmetric in $\alpha\beta$, and furthermore $\gamma_{\alpha\beta}^{M_1\cdots M_6}$ is self dual. Therefore, in 12 dimensions, on the right hand side of (1) there can be no P_M, and the 528 generators consist of $Z_{M_1M_2}$, and the self dual $Z^+_{M_1\cdots M_6}$. The number of components in each is $\frac{12\times 11}{2}=66$ and $\frac{1}{2}\frac{12\times11\times10\times9\times8\times7}{1\times2\times3\times4\times5\times6}=462$ respectively. Upon compactification to (10,1) we rewrite the 12D index $M=(0',\mu)$ where $\mu=0,1,2,\cdots,10$ is an 11D index. Then we have (suppressing the $0'$ index)

$$Z_{M_1M_2} \quad\rightarrow\quad P_\mu \oplus Z_{\mu_1\mu_2} \quad 66=11+55 \tag{2}$$

$$Z^{+}_{M_1\cdots M_6} \quad\rightarrow\quad X_{\mu_1\cdots\mu_5} \qquad\qquad 462 = 462$$

which are the momenta and central charges in 11 dimensions pointed out in [8].

Continuing the compactification process to lower dimensions on $R^{d-1,1}\otimes T^{c+1,1}$, each eleven dimensional index μ decomposes into $\mu\oplus m$ where μ is in d dimensions and m is in $c+1=11-d$ dimensions. Then each 11 dimensional tensor decomposes as follows

$$\begin{array}{rcl} P_\mu & \rightarrow & P_\mu\oplus P_m \\ Z_{\mu\nu} & \rightarrow & Z_{\mu\nu}\oplus Z_\mu^n\oplus Z^{mn} \\ X_{\mu_1\cdots\mu_5} & \rightarrow & X_{\mu_1\cdots\mu_5}\oplus X^{m_1}_{\mu_1\cdots\mu_4}\oplus X^{m_1m_2}_{\mu_1\mu_2\mu_3} \\ & & \oplus X^{m_1m_2m_3}_{\mu_1\mu_2}\oplus X^{m_1\cdots m_4}_{\mu_1}\oplus X^{m_1\cdots m_5}. \end{array} \tag{3}$$

For example in $d=10$ the type IIA superalgebra is recovered, with the 528 operators $(P_\mu, P_{10}, Z_{\mu\nu}, Z_\mu, X_{\mu_1\cdots\mu_4}, X^{\pm}_{\mu_1\cdots\mu_5})$ where the $\pm$ indicate self/antiself dual respectively. In Table I in each row labelled by $(d-1,1)/(c+1,1)$ the numbers of each central extension of P,Z,X type with p Lorentz indices is indicated (these are the numbers that are not in bold type). As we go to lower dimensions one must use the duality between p indices and $d-p$ indices to reclassify and count the central extensions $Z^{ab}_{\mu_1\cdots\mu_p}$. In the table a number in parenthesis means that it should be omitted from there and instead moved in the same row to the location where the same number appears in brackets . This corresponds to the equivalence of p indices and $d-p$ indices. When $p=d-p$ there are self-dual or anti-self-dual tensors. Their numbers are indicated with additional superscripts $\pm$ in the form $1^{\pm}, 2^{\pm}, 3^{\pm}, 10^{\pm}, 35^{\pm}$ wherever they occur.

The total number of central extensions P,Z,X found according to this compactification procedure for each value of p are indicated in Table-I in bold characters. These totals are the same numbers found by counting the *number* of possibilities ab on $Z^{ab}_{\mu_1\cdots\mu_p}$. The bold numbers following the $=$ sign correspond to representations of $SO(c+1,1)$ (making a connection to 12D) and those following the $\approx$ sign correspond to representations of K (to be discussed later in connection to duality).

$\frac{c+1,1}{d-1,1}$	32 Q^a_α $SO(c+1,1)$ (or K) $\otimes$ $SO(d-1,1)$	p=0 $\dot{P}^m$,Z^{mn} X^{mnlqr}	p=1 $\dot{P}_\mu$,Z^n_μ X^{nlqr}_μ	p=2 $\dot{Z}_{\mu v}$ $X^{lqr}_{\mu\nu}$	p=3 $\dot{X}^{qr}_{\mu\nu\lambda}$	p=4 $\dot{X}^{r}_{\mu_1 \cdot \mu_4}$	p=5 $\dot{X}_{\mu_1 \cdot \mu_5}$	$\frac{U}{K}$
A $\frac{1,1}{9,1}$	$(\pm,\mathbf{16})$	1+0 +0	1+1 +0	1 +0	0	1	1^+ $+1^-$	$\frac{SO(1,1)}{Z_2}$
B $\frac{1,1}{9,1}$	$\left({}^{+}_{+},\mathbf{16}\right)$	0+0 +0	1+2 +0	0 +0	1	0	1^+ $+2^+$	$\frac{SL(2,R)}{SO(2)}$
$\frac{2,1}{8,1}$	$(\mathbf{2},\mathbf{16})$	2+1 +0 $=\mathbf{3}$ $\approx\mathbf{2+1}$	1+2 +0 $=\mathbf{3}$ $\approx\mathbf{2+1}$	1+0 $=\mathbf{1}$ $\approx\mathbf{1}$	1	[1] +2 $=\mathbf{3}$ $\approx\mathbf{2}$ $+\mathbf{1}$	(1) *move*	$\frac{SL(2)\otimes SO(1,1)}{SO(2)\otimes Z_2}$
$\frac{3,1}{7,1}$	$\left((\mathbf{2},0),\mathbf{8}^+\right)$ $\left((0,\mathbf{2}),\mathbf{8}^-\right)$	3+3 +0 $=\mathbf{6}$ $\approx\mathbf{3}^+$ $+\mathbf{3}^-$	1+3 +0 $=(\mathbf{2},\mathbf{2})$ $\approx\mathbf{3+1}$	1+1 $=\mathbf{1+1}$ $\approx\mathbf{1+1}$	3+[1] $=(\mathbf{2},\mathbf{2})$ $\approx\mathbf{3+1}$	3^+ $+3^-$ $=\mathbf{6}$ $\approx\mathbf{3}^+$ $+\mathbf{3}^-$	(1) *move*	$\frac{SL(3)\otimes SL(2)}{SO(3)\otimes U(1)}$
$\frac{4,1}{6,1}$	$(\mathbf{4},\mathbf{8})$	4+6 +0 $=\mathbf{10}$ $\approx\mathbf{10}$	1+4 +1 $=\mathbf{5+1}$ $\approx\mathbf{5+1}$	1+4 +[1] $=\mathbf{5+1}$ $\approx\mathbf{5+1}$	6 +[4] $=\mathbf{10}$ $\approx\mathbf{10}$	(4) *move*	(1) *move*	$\frac{SL(5)}{SO(5)}$
$\frac{5,1}{5,1}$	$(\mathbf{4},\mathbf{4}^*)$ $(\mathbf{4}^*,\mathbf{4})$	5+10 +1 $=\mathbf{1+15}$ $\approx(\mathbf{4},\mathbf{4})$	1+5+ 5+[1] $=2\times\mathbf{6}$ $\approx(\mathbf{0},\mathbf{5})$ $+(\mathbf{5},\mathbf{0})$ $+2(\mathbf{0},\mathbf{0})$	1+10 +[5] $=\mathbf{1+15}$ $=(\mathbf{4},\mathbf{4})$	10^+ $+10^-$ $=\mathbf{10}^+$ $+\mathbf{10}^-$ $\approx(\mathbf{10},1)$ $+(1,\mathbf{10})$	(5) *move*	(1) *move*	$\frac{SO(5,5)}{SO(5)\otimes SO(5)}$
$\frac{6,1}{4,1}$	$(\mathbf{8},\mathbf{4})$	6+15 +6 +[1] $=\mathbf{7+21}$ $\approx\mathbf{27+1}$	1+6 +15+[6] $=\mathbf{7+21}$ $\approx\mathbf{27+1}$	1+20 +[15] $=\mathbf{1+35}$ $\approx\mathbf{36}$	(15) *move*	(6) *move*	(1) *move*	$\frac{E_{6(6)}}{USp(8)}$
$\frac{7,1}{3,1}$	$(\mathbf{8}^+,(\mathbf{2},\mathbf{0}))$ $(\mathbf{8}^-,(0,\mathbf{2}))$	7+21 +21 +[7] $=\mathbf{28+28}$ $\approx\mathbf{28}_c$	1+7 +35 +[21] $=\mathbf{8+56}$ $\approx\mathbf{63+1}$	$1^\pm$ $+35^\pm$ $=\mathbf{1}^\pm$ $+\mathbf{35}^\pm$ $\approx\mathbf{36}_c$	(21) *move*	(7) *move*	0	$\frac{E_{7(7)}}{SU(8)}$
$\frac{8,1}{2,1}$	$(\mathbf{16},\mathbf{2})$	8+28 +56 +[28] $=\mathbf{36+84}$ $\approx\mathbf{120}$	1+8+70 +[1+56] $=\mathbf{1+9}$ $+\mathbf{126}$ $\approx\mathbf{135}$ $+\mathbf{1}$	(1+56) *move*	(28) *move*	0	0	$\frac{E_{8(8)}}{SO(16)}$

Table I. Classification of Q^a_α and $Z^{ab}_{\mu_1\cdots\mu_p}$ under 11D (or 12D) and K.

What is the meaning of the p-form central extension $Z^{ab}_{\mu_1\cdots\mu_p}$? Since this is a charge in a global algebra, there ought to exist a $(p+1)$-form local

current $J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x)$ whose integral over a space-like surface embedded in d-dimensions gives

$$Z^{ab}_{\mu_1\cdots\mu_p} = \int d^{d-1}\Sigma^{\mu_0}\ J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x)\,. \tag{4}$$

The current couples to the fields of low energy physics (i.e. supergravity). In the case of usual central charges that are Lorentz singlets Z^{ab} (i.e. $p=0$) the current is associated with charged particles. Such a current may be constructed as usual from worldlines (or equivalently from local fields) as follows

$$J^{ab}_{\mu}(x) = \int d\tau \sum_i z^{ab}_i\, \delta^d\left(x - X^i(\tau)\right)\ \partial_\tau X^i_\mu(\tau)\,. \tag{5}$$

The z^{ab}_i are the charges of the particles labelled by i. This current couples in the action to a gauge field A^μ_{ab}, and it appears as the source in the equation of motion of the gauge field

$$\begin{array}{l} S \sim \sum_i \int d\tau\, A^\mu_{ab}\left(X^i(\tau)\right)\, \partial_\tau X^i_\mu(\tau)\ \ z^{ab}_i \\ \quad = \int d^d x\ A^\mu_{ab}(x)\ J^{ab}_\mu(x) \\ \partial_\lambda\, \partial^{[\lambda} A^{\mu]}_{ab}(x) = J^\mu_{ab}(x)\,. \end{array} \tag{6}$$

The generalization to the higher values of p is straightforward: In order to have a charge that is a p-form we need a current $J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x)$ that is a $(p+1)$-form. This in turn requires a p-brane to construct the current,

$$\begin{aligned} J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x) &= \int d\tau d\sigma_1 ... d\sigma_p \sum_i z^{ab}_i\, \delta^d\left(x - X^i(\tau,\sigma_{1,}\cdots\sigma_p)\right) \qquad (7)\\ &\quad \times \partial_\tau X^i_{[\mu_0}\cdots\partial_{\sigma_p} X^i_{\mu_p]}(\tau,\sigma_{1,}\cdots\sigma_p)\ , \end{aligned}$$

and its coupling to supergravity fields requires a $(p+1)$-form gauge potential $A_{\mu_0\mu_1\cdots\mu_p}(x)$ such that

$$\begin{aligned} S &\sim \int d^d x\ A^{\mu_0\mu_1\cdots\mu_p}_{ab}(x)\ J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x) \qquad (8)\\ &= \sum_i \int d\tau d\sigma_1 ... d\sigma_p\, A^{\mu_0\mu_1\cdots\mu_p}_{ab}\left(X^i\right)\ \partial_\tau X^i_{[\mu_0}\cdots\partial_{\sigma_p} X^i_{\mu_p]}\ z^{ab}_i, \end{aligned}$$

and

$$\partial_\lambda\, \partial^{[\lambda} A^{\mu_0\mu_1\cdots\mu_p]}_{ab}(x) = J^{\mu_0\mu_1\cdots\mu_p}_{ab}(x)\,. \tag{9}$$

As is well known by now there are perturbative as well as non-perturbative couplings of p-branes to supergravity in various dimensions. Hence the $Z^{ab}_{\mu_1\cdots\mu_p}$ are present in the superalgebra and they correspond simply to the charges of p-branes. The classification of their ab indices under duality groups is the subject of the next section, but here we already see that there is a one to one correspondence between the p-forms $Z^{ab}_{\mu_1\cdots\mu_p}$ and the $(p+1)$-form gauge potentials $A_{ab}^{\mu_0\mu_1\cdots\mu_p}$ that appear as massless states in string theory in the NS-NS or R-R sectors.

The main message is that from the point of view of the superalgebra all p-branes appear to be at an equal footing. Isometries of the superalgebra that will be discussed below treats them equally and may mix them with each other in various compactifications. The theory in d dimensions has $(p+1)$-forms $A_{ab}^{\mu_0\mu_1\cdots\mu_p}$ which appear as massless vector particles in the string version of the fundamental theory. These act as gauge potentials and couple at low energies to charged p-branes. This generates a non-trivial central extension $Z^{ab}_{\mu_1\cdots\mu_p}$ in the superalgebra. The number of such central extensions (ab indices) is in one-to-one correspondence with the number of the $(p+1)$-forms $A_{ab}^{\mu_0\mu_1\cdots\mu_p}$, and these numbers can be obtained by counting the possible combination of (symmetric/antisymmetric) indices ab associated with the supercharges.

3 Duality groups

In the discussion above we concentrated on the 11D (or 12D) content of the supercharges and the central extensions. We now turn to duality. In string theory the T-duality group is directly related to the number of compactified left/right string dimensions. Therefore, in our notation, for a string of type II it is $T = SO(c,c)$. Its maximal compact subgroup is $k = SO(c)_L \otimes SO(c)_R$ where L, R denote left/right movers respectively. The index a on the supercharges Q^a_α corresponds precisely to the spinor index of $SO(c)_L \otimes SO(c)_R$ (see table III in [12]). Investigating the supercharges listed in Table I shows that the index a that was classified there under the hidden non-compact group $SO(c+1,1)_{hidden}$ can be reclassified under the perturbatively explicit maximal compact subgroup $k \subset T$ of T-duality, $k = SO(c)_L \otimes SO(c)_R$. These two groups are not subgroups of each other, but they do have a common subgroup $SO(c)$. Recall that c is the number of compactified dimensions (other

than the two hidden dimensions), and $SO(c)$ is the rotation group in these internal dimensions.

Next we look for the *compact* group K that contains $SO(c)_L \times SO(c)_R$, $SO(c+1)$ and that has an *irreducible* representation for the index a (total dimension N). By virtue of containing $k \subset T$ the group $K \supset k$ must be related to a larger group of duality U that contains T. The groups K and U are listed in Table I. The subgroup hierarchy that emerges is as follows

$$
\begin{array}{ccccc}
\underset{c\ compact+2\ hidden\ dims.}{SO(c+1,1)} \otimes \underset{spacetime}{SO(d-1,1)} & \rightarrow & \begin{array}{c} a \text{ on } Q^a_\alpha \\ ab \text{ on } Z^{ab}_{\mu_1\cdots\mu_p} \end{array} & & \\
\downarrow & & \uparrow & & \\
\left.\begin{array}{c} SO(c+1)\ {}_{1\,hidden\,dim} \\ SO(c)_L \otimes SO(c)_R \\ \uparrow \\ \underset{(T-duality)}{SO(c,c)} \end{array}\right\} & \Longleftarrow & \left.\begin{array}{c} \overset{maximal\ compact}{\underset{(duality)}{K}} \\ \\ \\ \end{array}\right\} & \Longleftarrow & \underset{(duality)}{U}
\end{array}
$$

Since the same N dimensional basis of supercharges labelled by a knows about both duality and the hidden dimensions, this must provide a bridge for relating properties of the states of the theory under both qualities. The first consequence of this is the reclassification of the central extensions $Z^{ab}_{\mu_1\cdots\mu_p}$. Previously they were related to 11D (or 12D) as in Table I. But now the combination ab corresponds to the symmetric or antisymmetric product of the N dimensional representation of K. Therefore, *the central extensions are now also classified under* $\dot{K}$. The result is the total dimension listed in Table I in bold numbers following the $\approx$ sign. These numbers are indeed dimensions of irreducible multiplets under K.

The main point is that the supercharges as well as the central extensions are now classified under hidden (broken) symmetries of two different types. The first one $SO(c+1,1)_{hidden}$ relates to 11 or perhaps 12 hidden dimensions, and the second one $K \subset U$ relates to U-duality. The common compact subgroup $SO(c+1)$ already contains non-perturbative information about the spacelike hidden dimension, but more information about the hidden time-like dimension and about U-duality is contained in the larger group structures.

4 U-duality and non-perturbative states

Under the assumption that the superalgebra is valid as a dynamical (broken) symmetry in the entire theory, all states would belong to multiplets of the (broken) superalgebra, including the central extensions and the p-branes associated with them. One would then expect to be able to classify the physical states of the theory according to different modules of $SO(c+1,1)_{hidden}$ and $K \subset U$ that have intersections with each other in the form of (broken) $SO(c+1)$ multiplets. Each one of these classifications contains non-perturbative states related to either duality or hidden dimensions. By finding them and studying their couplings consistent with the superalgebra one would be able to learn certain global properties of the underlying theory.

The scheme for finding the non-perturbative states is as follows. First identify the perturbative string states, classify them under supermultiplets and identify their classification under the perturbatively explicit $SO(c)_L \otimes SO(c)_R$. Then try to reclassify them under the bigger group K. If additional states are needed to make complete K multiplets add them (these extra states are presumably p-branes, D-branes). There may be non-unique ways of completing K multiplets. If so, then try to make it consistent with the presence of the hidden dimensions by making sure that the $SO(c{+}1)$ representations embedded in K multiplets are consistent with the structure of the central charges listed in the table. When this is achieved one should also check that it is all consistent with a compactification of a collection of states that starts in 11 dimensions, i.e. consistency with 11-dimensional (broken) multiplets with signature (10,1). One may need to add at this stage more non-perturbative states that are not in the same K-multiplet with some perturbative string state (presubambly more p-or D-brane states). So far one should expect consistency with “M-theory”. Finally, check if the structure of the representations that emerge in this way is also consistent with 12 dimensions, with signature (10,2). In this way many properties of non-perturbative states can be deduced.

Such a program was initiated in previous papers [11] [12] [14]. The only central extension included in those discussions is the $p=0$ case in various dimensions. Recall that the $p=0$ central extension in lower dimensions contains pieces of the $p \geq 1$ central extensions of higher dimensions. Therefore, the non-perturbative states include p-branes in their internal dimensions. Their results, on the consistency between (10,1) and U-duality is concerned,

are summarized here. More will be said elsewhere [10] about (10,2) and the other central extensions.

5 U-duality and 11D

5.1 perturbative and non-perturbative states

In the toroidaly compactified type II string on $R^{d-1,1} \otimes T^c$, with $d + c = 10$, the perturbative vacuum state has Kaluza-Klein (KK) and winding numbers, and is also labelled by the $2^7_B + 2^7_F$ dimensional Clifford vacuum of zero modes (in the Green-Schwarz formalism). The closed string condition $L_0 = \bar{L}_0$ can be satisfied without requiring equal excitation levels $l_{L,R}$ for left/right movers. Hence the perturbative states are

$$\begin{array}{l} (\mathrm{Bose} \oplus \mathrm{Fermi\ oscillators})_{\mathrm{L}}^{(\mathrm{l_L})} \\ \times (\mathrm{Bose} \oplus \mathrm{Fermi\ oscillators})_{\mathrm{R}}^{(\mathrm{l_R})} \\ \times\ |vac,\ p^\mu; \vec{m}, \vec{n} > \end{array} \tag{10}$$

where the c-dimensional vectors $(\vec{m}, \vec{n})$ are the Kaluza-Klein and winding numbers that label the "*perturbative base*". These quantum numbers satisfy the relations

$$\begin{aligned} l_L + \frac{1}{2}\vec{p}_L^2 &= l_R + \frac{1}{2}\vec{p}_R^2 = M_d^2 \\ \vec{p}_R^2 - \vec{p}_L^2 &= \vec{m} \cdot \vec{n} = l_L - l_R \end{aligned} \tag{11}$$

where $\vec{p}_{L,R}$ depend as usual [16] on $(\vec{m}, \vec{n})$ and (G_{ij}, B_{ij}) that parametrize the torus T^c, while M_d is the mass in d-dimensions $M_d^2 = p_\mu^2$. By using the methods of [11] we can identify the following supermultiplet structure for the string states (10) at levels (l_L, l_R)

$$\begin{array}{rcl} (0,0) & : & \left(2^7_B + 2^7_F\right) \otimes 1_L \otimes 1_R \\ (0, l_R) & : & \left(2^{11}_B + 2^{11}_F\right) \otimes 1_L \otimes \sum_i r_{iR}^{(l_R)} \\ (l_L, 0) & : & \left(2^{11}_B + 2^{11}_F\right) \otimes \sum_i r_{iL}^{(l_L)} \otimes 1_R \\ (l_L, l_R) & : & \left(2^{15}_B + 2^{15}_F\right) \otimes \sum_i r_{iL}^{(l_L)} \otimes \sum_i r_{iR}^{(l_R)} \end{array} \tag{12}$$

The $2_B^{11}+2_F^{11}$ corresponds to the intermediate supermultiplet of 11D supersymmetry. The structures $\sum_i r_{iL,R}^{(l_{L,R})}$ are listed in Table II up to level 5.

Level	SO(9)$_{L,R}$ reps $\left(\sum_i r_i^{(l_{L,R})}\right)_{L,R}$
$l_{L,R}=1$	1_B
$l_{L,R}=2$	9_B
$l_{L,R}=3$	44_B+16_F
$l_{L,R}=4$	$(9+36+156)_B+128_F$
$l_{L,R}=5$	$\begin{pmatrix} 1+36+44+84 \\ +231+450 \end{pmatrix}_B + [16+128+576]_F$

Table II. L/R oscillator states in 10D.

The $SO(9)_{L,R}$ representations in this table are reduced to representations of $SO(d-1)_{L,R}\otimes SO(c)_{L,R}$. So, a general perturbative string state is identified by "index space" and "base space" in the form

$$\phi_{indices}^{(l_L l_R)}(base) \tag{13}$$

The base space are the quantum numbers coming through the $(\vec{m},\vec{n})$ and the indices are given by the product of representations in (12) and Table II (which may be extended beyond level 5). These are all the perturbative type II string states in d-dimensions.

The spectrum of the non-perturbative states is much richer. There are many central charges in the supersymmetry algebra (see Table I) and those provide sources that couple to the NS-NS as well as R-R gauge potentials. Therefore one finds a bewildering variety of non-perturbative solutions of the low energy field equations as examples of non-perturbative states that carry the non-perturbative p-brane charges $Z_{\mu_1\cdots\mu_p}^{ab}$. We will take an algebraic approach to describe them, by imposing the stucture of the superalgebra discussed earlier. The base quantum numbers are now extended to include the non-perturbative charges that appear in the global superalgebra (here we concentrate on p =0-branes only, ignoring the higher p-branes in this paper).

$$|vac,\ p^\mu;\vec{m},\vec{n},z^I > \tag{14}$$

where the 0-brane charges are $\left(\vec{m},\vec{n},z^I\right)$. From the point of view of string theory, the z^I are non-perturbative charges that couple to the R-R sector,

while $(\vec{m},\vec{n})$ are the perturbative charges that couple to the NS-NS sector. In the notation of Table I we identify the generators that correspond to $\left(\vec{m},\vec{n},z^I\right)$ as follows

$$\begin{aligned} \vec{m} &\rightarrow P^i, \quad \vec{n} \rightarrow Z^{i,c+1}, \quad z^I \rightarrow (P^{c+1}, Z^{ij}, X^{r_1\cdots r_5}) \\ i,j &= 1,2,\cdots,c \qquad r_1 = 1,\cdots,c,c+1 \end{aligned}$$

That is, $\vec{m}$ corresponds to the Kaluza-Klein momenta excluding the extra hidden coordinate, the winding numbers $\vec{n}$ correspond to the last column or row of $Z^{r_1r_2}$, while all remaining 0-brane charges are non-perturbative. Although there is a big asymmetry among these charges from the point of view of the string, they are on equal footing from the point of view of the superalgebra, and they are classified in higher multiplets of $SO(c+1,1)_{hidden}$ and of $K\subset U$ as discussed before. The multiplets $Z^{ab}=\left(\vec{m},\vec{n},z^I\right)$ form the *non-perturbative base* in $\phi_{indices}\,(base)$ [1].

There are two types of new non-perturbative states: those obtained by applying string oscillators on the non-perturbative base and those that cannot be obtained in this way, but which are required to be present to form a basis for U-duality transformations. The second kind require the extension of the indices such that complete K multiplets are obtained. These are needed as intermediate states in matrix elements of the superalgebra which is assumed to be valid in the full theory. So, a general state in the theory is identified at each $l_{L,R}$ as in (13). Both the base and the indices have non-perturbative extensions. The full set of states is required to form a basis for U-duality transformations at each fixed value of $l_{L,R}$. These states are not degenerate in mass, hence the idea of a multiplet is analogous to the multiplets in a theory with broken symmetry.

The BPS saturated states are those with either $l_L=0$ or $l_R=0$. Even for BPS saturated states there are the two types of non-perturbative states. Typically the non-perturbative indices occur for $l_L\geq 2,\ l_R=0$. For the

[1] According to the dimensions of representations in Table I, the 0-brane $Z^{ab}=\left(\vec{m},\vec{n},z^I\right)$ seem to correspond to complete linear representations of U for all dimensions except for $d=3$ (when $U=E_{8(8)}$). Similarly, higher p-branes $Z^{ab}_{\mu_1\cdots\mu_p}$ do not generally form linear representations of U. Also they seem to form complete representations of $SO(c,c)$ for all cases except for $(d=5,\,p=3),(d=3,4,\,p=2)$. We interpret these observations to mean that the base is not generally a *linear* representation of either T or U duality groups, but it is a *linear* representation of K or $SO(c+1,1)$.

BPS saturated states one can derive an exact non-perturbative formula for the mass by using the supersymmetry algebra with central charges. For example for $(d = 9, \quad c = 1)$ for a non-perturbative BPS state with KK momentum $p_9 = m/R$, winding $w = nR$ and non-perturbative eleventh momentum $p_{c+1} = z/r$, we have $l_R = 0$, $l_L = mn$, and a mass formula

$$M = \frac{1}{\sqrt{2}} |nR_{10} + \sqrt{m^2/R^2 + z^2/r^2}|. \tag{15}$$

where m, n, z are quantized integers and R, r are moduli. The presence of the non-perturbative (quantized) p_{c+1} is a new piece in the mass formula that differs from the perturbative string BPS states. This formula is derived from the superalgebra with the usual methods, but allowing for the non-perturbative p_{c+1}. A special case is the uncompactified theory in 10D, for which the BPS states (called black holes in [3]) have masses proportional to the 11th momentum. Further generalizations involving other non-trivial z^I will be given elsewhere. For non BPS saturated states we cannot give an exact mass formula.

What about the hidden dimensions? In the *uncompactified* theory consider all the states, including their values of the non-perturbative 11th momentum . In Fourrier space the fields $\phi_{indices}\,(x^\mu, x^{11})$ seem to be 11-dimensional. This is possible only if the indices also have an 11D structure. At levels $l = 0, 1$ it has been known that this is true for a long time for the usual string states, and this is evident from Table II (level $l = 1$ is a singlet times the factor $2_B^{15} + 2_F^{15}$ which has 11D content). At higher levels $l \geq 2$ the string states by themselves do not have the 11D structure for the indices. The minimal structure of indices that would be needed in an 11D theory was identified for all levels. This minimal structure has a definite pattern for massive states given by

$$indices \Rightarrow \left(2_B^{15} + 2_F^{15}\right) \times R^{(l)}. \tag{16}$$

The factor $2_B^{15} + 2_F^{15}$ can be interpreted as the action of 32 supercharges on a set of $SO(10)$ representations $R^{(l)}$ at oscillator level l. For the minimal set of indices the factor $R^{(l)}$ is of the form of a sum of SO(9) representations that make up SO(10) representations.

$$R^{(l)} = \sum_{l'=1}^{l} \left(\sum_i r_i^{(l')} \right)_L \times \sum_{l'=1}^{l} \left(\sum_i r_i^{(l')} \right)_R \tag{17}$$

Each term in the sum over l' looks like the string states in Table II [11]. Only the highest term ($l' = l$) corresponds to the perturbative string states of level l. The remaining terms correspond to non-perturbative states with quantum numbers isomorphic to those listed in Table II at the given levels. The meaning of this pattern has not been understood so far. Furthermore, in the complete theory there may be more states beyond the minimal set displayed above.

What about 12D? Can the states discussed above also be classified under $SO(10,2)$. First, for $l = 0$ the answer is yes, since the massless states classified with the Poincare group are also a representation of the conformal group. Then the 11D $2_B^7 + 2_F^7$ massless states classified according to the Lorentz group SO(10,1) also form a basis consistent with the conformal group SO(10,2). A more interesting case is the $l = 1$ first massive level states $2_B^{15} + 2_F^{15}$. As mentioned above, at rest the physical states come in complete $SO(10)$ multiplets, where $SO(10)$ is the rotation group in 11-dimensions. From the point of view of $SO(10,2)$ we would like to show that they come in complete multiplets of $SO(10,1)'$ where the time like component is the hidden timelike coordinate. Indeed this is true for the $2_B^{15} + 2_F^{15}$ states! This can be explained as being a simple property of the first excited level and of the supercharges, as follows: These states may be regarded as the simplest massive supermultiplet created by applying all possible combinations of the 32 supercharges on a singlet vacuum. Since the vacuum is a singlet of $SO(10,1)'$, and the supercharges form the spinor representation of $SO(10,1)'$ (by virtue of being a representation of $SO(10,2)$), then the classification of the states under $SO(10,1)'$ follows automatically from the products of the 32-dimensional spinor. This is an interesting signal of the presence of a hidden timelike dimesion. In this paper there will be no more discussion of higher excited levels from the point of view of 12D

5.2 Dualities and non-perturbative spectrum

The perturbative string states involved in the T-duality transformations are not all degenerate in mass. Therefore, T-duality must be regarded as the analog of a spontaneously broken symmetry, and the string states must come in complete multiplets despite the broken nature of the symmetry. It is well known that $T = O(c,c;Z)$ acts linearly on the the $2c$ dimensional vector $(\vec{m},\vec{n})$. However it is important to realize that it also acts on the indices

of $\phi_{indices}$ in definite representations [12]. The action of T on the indices is an induced k-transformation that depends not only on *all* the parameters in T but also on the background $c \times c$ matrices (G_{ij}, B_{ij}) that define the tori T^c . Since the states in the previous section are all in $k = O(c)_L \times O(c)_R$ multiplets, the T-duality transformations do not mix perturbative states with non-perturbative states.

A U-multiplet contains both perturbative as well as non-perturbative T-multiplets. Like the T-duality transformations, the U-duality transformations act *separately* on the base and the indices of the states described by (13) *without mixing index and base spaces.* The action of U on the base quantum numbers $(\vec{m}, \vec{n}, z_I)$ is a linear transformation in a representation of same dimension as the representation of K listed in Table I [12]. The action on index space is an *induced field-dependent gauge transformation in the maximal compact subgroup K*, whose only free parameters are the global parameters in U. This (U, K) structure extends the situation with the (T, k) structure of the T-duality transformations described in the previous paragraph. The logical/mathematical basis for this structure is induced representation theory. The bottom line is that in order to have U-duality multiplets, in addition to the non-perturbative base, *the "indices" on the fields in (13) must form complete K-multiplets.*

By knowing the structure of a U-multiplet we can therefore predict algebraically the quantum numbers of the non-perturbative states by extending the quantum numbers of the known perturbative states given in (12). The prediction of these non-perturbative quantum numbers is one of the immediate outcomes of our approach.

5.3 An example

It is very easy to analyze the case $(d, c) = (6, 4)$ so we present it here as an illustration. In this case the spin group is $SO(5)$ and there are 4 internal dimensions. The duality groups and index spaces follow from Tables I,II and

(12). The relevant information is summarized by

$$\begin{array}{l} U = SO(5,5), \quad K = SO\left(5\right) \otimes SO\left(5\right) \\ T = SO(4,4), \quad k = SO\left(4\right)_L \otimes SO\left(4\right)_R \\ l_{L,R} = 1: \quad \left(\sum_i r_i^{(l_{L,R})}\right)_{L,R} = 1_{L,R} \\ l_{L,R} = 2: \quad \left(\sum_i r_i^{(l_{L,R})}\right)_{L,R} = 9_{L,R} \\ \qquad\qquad\qquad = 5_{L,R}^{space} \oplus 4_{L,R}^{internal} \\ l_{L,R} = 3: \quad etc. \end{array} \tag{18}$$

where the $9_{L,R}$ have been reclassified according to their space and internal components. The reclassification is done also for the short $(2_B^7 + 2_F^7)$, intermediate $(2_B^{11} + 2_F^{11})$ and long $(2_B^{15} + 2_F^{15})$ supermultiplet factors. It is clear from this form that the $k = SO\left(4\right)_L \otimes SO\left(4\right)_R$ structure follows directly from the separate left/right internal components, while the spin of the state is to be obtained by *combining* left and right content of the space part.

Here I will discuss an example involving BPS states which is very similar to another discussion on non-BPS states given in [12]. Let us consider the BPS saturated states $(l_L \neq 0,\ l_R = 0)$. The base quantum numbers in $\phi_{indices}^{(l_L,0)}(base)$ form the 16 dimensional spinor representation of $U = SO(5,5)$

$$base = \left(\vec{m}, \vec{n}, z^I\right) = 16 \quad \text{of} \quad \mathrm{SO}(5,5) \tag{19}$$

Among these the eight quantum numbers $(\vec{m}, \vec{n})$ are perturbative, while the remaining eight z^I are non-perturbative. 0-branes that carry these quantum numbers provide the sources for the field equations of the 8 massless NS-NS vectors and the 8 R-R vectors respectively. The representation content of the indices in $\phi_{indices}^{(l_L,0)}(base)$ is

$$\begin{array}{l} indices = (2_B^{11} + 2_F^{11}) \times \\ \times \left[\begin{array}{l} \left(\sum_i r_i^{(l_L)}\right)_L \\ +non-perturbative \end{array} \right] \end{array} \tag{20}$$

where $(2_B^{11}+2_F^{11})$ is interpreted as the SUSY factor. The full set of indices must form complete $K = SO\left(5\right)_L \otimes SO\left(5\right)_R$ multiplets for consistency with the general U-duality transformation. It can be shown generally that the SUSY factor does satisfy this requirement because the supercharges themselves are representations of $SO(5)_{spin} \times K$ [12]. Therefore, the remaining factor in brackets must be required to be complete $SO(5)_{spin} \times K$ multiplets.

At level $l_L = 1$ the piece $\sum_i r_i^{(1)} = 1$ is a singlet, as seen in Table II. Hence no additional non-perturbative indices are needed at this level. At level $l_L = 2$ the piece $\sum_i r_i^{(2)} = 9_L = 5_L^{space} \oplus 4_L^{internal}$ is classified under $SO(5)_{spin} \times SO(4)_L \otimes SO(4)_R$ as

$$(5,(0,0)) + (0,(4,0)) \,. \tag{21}$$

Obviously, this is not a complete $SO(5)_{spin} \times SO(5)_L \otimes SO(5)_R$ multiplet. Therefore, non-perturbative indices must be added just in such a way as to extend the $(4,0)$ of $k = SO(4)_L \otimes SO(4)_R$ into the $(5,0)$ of $K = SO(5)_L \otimes SO(5)_R$. That is

$$(4_{int})_L \rightarrow (5_{int})_L \,. \tag{22}$$

This extension determines the required non-perturbative indices for this case. Note that this amounts to extending the 9_L into a 10_L, and similarly for right-movers

$$9_{L,R} \rightarrow 10_{L,R}. \tag{23}$$

This is precisely what was needed in section-1 in order to obtain consistency with an underlying 11D theory [11].

At all higher levels $l_{L,R}$ the requirement for complete $K-$multiplets coincides precisely with the requirement of an underlying 11D theory. Therefore the full set of indices are the same as those given in eq.(17). The story is the same with the non-BPS-saturated states at arbitrary $l_{L,R}$. This result was found in [11] by assuming the presence of hidden 11-dimensional structure in the non-perturbative type-IIA superstring theory in 10D. In ref.[11] a justification for (23) could not be given. However, in [12] and in the present analysis U-duality demands (22) and therefore justifies (23), and similarly for all higher levels.

Therefore for this particular compactification on $R^6 \otimes T^4$, U-duality and 11D Lorentz representations imply each other.

A consistency check between U-duality and D-branes was reported in [14] and in this conference. It is of interest to compare that analysis to ours. We find complete agreement at level $l_L = 1$. But at higher levels $l \geq 2$ our scheme requires more states than the D-brane degeneracy computed in [14]. In his case the states corresponding to the non-perturbative indices were not considered, seemingly because the special U-duality transformation he considered (interchanging the two 8's in the 16 of (19)) has a trivial

transformation on our index space (does not go outside of the $4_{int}^{L,R}$). We have seen that under more general U-transformations the extra indices are needed both for U-duality multiplets as well as for the 11D interpretation. Thus, the D-brane or other interpretation of these extra states is currently unknown.

For $(d,c) = (10,0)\,,(9,1)\,,(8,2)\,,(6,4)$ the analysis for $l_{L,R} = 2,3,4,5$ produces exactly the same conclusion as the 11D analysis. That is, U-duality demands that the $SO(9)_L \otimes SO(9)_R$ multiplets $\sum_i r_i^{(l_{L,R})}$ should be completed to $SO(10)_L \otimes SO(10)_R$ multiplets. The minimal completion (17) is sufficient in this case. Hence, in these compactifications U-duality is consistent with a hidden 11D structure, and in fact they imply each other.

On the other hand for the other values $(d,c) = (7,3)\,,(5,5)\,,(4,6)\,,(3,7)$ the story is more complicated. At various low levels we found that the minimal index structure required to satisfy U-duality is different than the *minimal structure* of 11-dimensional supersymmetry multiplets (17). If both U-duality and 11D are true then there must exist an even larger set of states such that they can be regrouped either as 11D multiplets or as U-duality multiplets. Exposing one structure may hide the other one. In fact we have shown how this works explicitly in an example in the case $(7,3)$ at low levels $l_{L,R}$ [12]. However, it is quite difficult to see if the required set of states can be found at all levels.

6 Final remarks

The basic assumption that we made is that the superalgebra is valid in the sense of a (broken) dynamical symmetry for the full theory. By studying the isometries of the superalgebra, including the central extensions, many of the features of duality could be displayed while some new features became apparent, including the following:

1. The central extensions (and the supercharges) have a structure consistent with two hidden spacetime dimensions, with an overall signature (10,2).

2. As a consequence of central extensions of the superalgebra, p-branes naturally become part of the fundamental theory, and their interaction with $p+1$ forms in supergravity are deduced. These p-branes

contribute to the non-perturbative states demanded by U-duality and hidden higher dimensions on an equal footing.

3. The structure of U-duality in type II superstrings, the groups, the non-perturbative states and their classifications emerge naturally from the structure of the superalgebra. This is summarized by Table I. Furthermore, one may start with perturbative string states, but then add non-perturbative states that are needed in order to provide a basis for the underlying superalgebra and its isometries. This is a method of finding at least some of the a-priori unknown non-perturbative states.

7 References

References

[1] K. Kikkawa-and M. Yamasaki, Phys.Lett. **149B** (1984) 357.

[2] K. Kikkawa-and M. Yamasaki, , Prog.Theor.Phys.**76** (1986) 1379.

[3] C. Hull and P. Townsend, Nucl. Phys. B438 (1995) 109.

[4] E. Witten, Nucl. Phys. B443 (1995) 85, and "Some comments on String Dynamics", hep-th/9507121, to appear in the proceedings of Strings '95.

[5] P.K. Townsend, Phys.Lett.**B350** (1995) 184. (= hep-th/9501068).

[6] J. Schwarz, Phys.Lett.**B367** (1996) 97 (=hep-th/9510086); hep-th/9509148; "M-theory extensions of T duality", hep-th/9601077.

[7] P. Horava and E. Witten, Nucl. Phys. **B460 (**1996) 506 (= hep-th/9510209).

[8] P. Townsend, "p-brane democracy", hep-th/9507048.

[9] J. Polchinski, Phys.Rev.Lett. 75 (1995) 4724 (= hep-th/9510017). See also, J. Polchinski, S. Chaudhuri, C.V. Johnson "NOtes on D-branes", hep-th/9602052.

[10] I. Bars, "Supersymmetry, p-brane duality and hidden spacetime dimensions", USC-96/HEP-B2 or hepth/9604139.

[11] I. Bars, Phys. Rev. **D52** (1995) 3567 (= hep-th/9503228).

[12] I. Bars and S. Yankielowicz, Phys. Rev. **D53** (1996) 4489 (=hep-th/9511098). I. Bars, "Consistency between 11D and U-duality", hep-th/9601164 .

[13] M. Blencowe and M. Duff, Nucl. Phys. **B310** (1988) 387.

[14] A. Sen, Nucl. Phys. B450 (1995) 103.

[15] R. Haag, J. Lopuszanski and M. Sohnius, Nucl.Phys.**B88** (1975) 257.

[16] See review, A. Giveon, M. Porrati and E. Rabinovici, Phys. Rep. 244 (1994) 77.